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SHORT COMMUNICATION

IN-PLANE FLOOR FLEXIBILITY EFFECTS ON TORSIONALLY UNBALANCED SYSTEMS

J. DE-LA-COLINA*
Facultad de Ingeniería, Universidad Autónoma del Estado de México, Toluca, México

SUMMARY

This paper presents the results of an analytical work addressed to understand the effects of in-plane floor flexibility on torsionally unbalanced (TU) systems subjected to bidirectional firm-soil earthquake records. The study uses a structural system consisting of a linear-elastic diaphragm supported by non-linear frames oriented along two orthogonal directions. The diaphragm is modelled with plane-stress finite elements and frames with stiffness-degrading flexural elements. Results indicate that an increase of in-plane diaphragm flexibility leads to a reduction of frame displacements for systems with initial lateral period of vibration $T > 0{\cdot}4$ s. For systems with $T \leqslant 0{\cdot}4$ s, in-plane floor flexibility can lead to significant frame displacement increments (50 per cent higher). Results show that these variations on displacements decrease for increasing values of both the seismic-force reduction factor and the system initial lateral period. Copyright © 1999 John Wiley & Sons Ltd.

KEY WORDS: torsion; bi-directional motion; flexible diaphragm; no-linear response; torsional coupling

INTRODUCTION

Floor stiffness plays an important role in distributing horizontal forces to lateral-resisting elements. In most cases, the hypothesis of a rigid diaphragm allows a significant reduction in the computational effort required in the structural analysis of buildings. Sometimes, however, structural configurations have large spans between lateral resisting elements making the assumption of rigid diaphragm questionable. Moreover, plans having irregular distributions of mass or stiffness can exacerbate the effect of floor flexibility due to the torsional unbalance of the structural system. In a torsionally unbalanced system the centre of mass and the initial centre of rigidity are not coincident.[1]

* Correspondence to: J. De-La-Colina, Facultad de Ingeniería, Universidad Autónoma del Estado de México, Apartado Postal 2-94, Toluca, 50000, México

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The vibration and the seismic response of systems with flexible diaphragms have been studied before.[2–9] Goldberg and Herness[2] studied torsionally balanced (TB) framed structures with linear-elastic diaphragms and lateral resisting elements. The influence of in-plane floor flexibility on the dynamic properties of symmetric (TB) linear-elastic buildings was also studied by Shepherd and Donald[3] who concluded that neglecting in-plane floor flexibility does not significantly change the computed dynamic properties of symmetric buildings. Jain[4] studied the dynamic properties of narrow symmetric (TB) linear-elastic buildings excited by unidirectional ground motions. He showed that, for long narrow buildings with identical frames and identical floors as well as equal masses lumped at the intersections of floors and frames, the vibration modes that include in-plane floor deformations are not excited by ground motion. Similar studies on linear-elastic narrow-plan TB buildings with two identical exterior frames were also reported by Jain and Jennings.[5]

Using non-linear models of the slab and the lateral resisting elements, Kunnath *et al.*[6] studied the effect of in-plane floor flexibility on the seismic response of narrow symmetric buildings with end walls. They showed that floor flexibility imposes larger demands (displacements and forces) on flexible frames. Their work was focused toward the development of an analytical tool to avoid the use of finite elements for modelling the slab. The study of Kunnath *et al.*,[6] however, did not cover explicitly the in-plane floor flexibility effect on the torsional seismic response of buildings.

Saffarini and Qudaimat[7] studied the error bounds that result from using the assumption of rigid diaphragms in buildings for several plan configurations. In their study, all analyses were linear-elastic, a restriction that is not applicable to many buildings excited by strong earthquakes. For framed buildings, they found that the use of a rigid in-plane floor hypothesis leads to almost the same results as those obtained with a flexible-floor hypothesis; however, for buildings containing shear walls, they concluded that error can result from the use of the flexible-floor hypothesis. Moreover, they found that the magnitude of the error varies depending on the ratio of the in-plane floor stiffness to the lateral-load resisting system stiffness. A similar study by Ju and Lin[8] considered linear-elastic TB buildings with shear walls. Their work was focused to obtain statistically a formula to estimate the difference in peak column forces between rigid-floor and flexible-floor analyses.

Torsionally unbalanced (TU) linear-elastic systems with flexible diaphragms were studied by Tena-Colunga and Abrams.[9] They concluded that torsion effects can be significantly reduced when in-plane floor flexibility increases. They also concluded, however, that diaphragm and shear-wall accelerations can increase with diaphragm flexibility in some cases. Recently, Yeomans and Dávila[10] studied the non-linear response of isolated diaphragms to develop a diaphragm hysteretic model.

It can be observed that most of the previous work uses linear-elastic lateral resisting elements and torsionally balanced systems. Simple reasoning suggests that in-plane floor flexibility increases the system lateral period as compared with that of a rigid-floor system. Therefore, if the lateral period of a system with flexible-floor is identified as the effective lateral period, it is not difficult to anticipate that the response of the flexible-floor system would be a function of this period and the shape of the response spectrum. However, torsional response could demand larger ductility on some lateral resisting elements of flexible-floor systems relative to rigid-floor systems. It is the objetive of this work to understand the effect of in-plane floor flexibility on torsionally unbalanced systems supported on non-linear elements and subjected to bidirectional earthquake records.

STRUCTURAL MODEL AND ANALYSIS FRAMEWORK

For the present study, a simple model consisting of a rectangular diaphragm supported by four frames is used (Figure 1). The diaphragm has plan dimensions a and b, with $b = 2a$. The Y-direction frame 1 is stiffer than frame 2 so that the initial centre of stiffness (CR) is located to the left of the floor geometric centre with an eccentricity $e_s = eb$. For a uniform distribution of mass on the diaphragm, the system is torsionally unbalanced. The X-direction frames are assumed to be identical so that the X axis is an axis of symmetry. The initial stiffness in each orthogonal direction is assumed to be the same, so that the vibration periods along both directions are equal, i.e., $T_x = T_y = T$.

For analysis purposes, the floor was idealized with 16 four-node plane-stress finite elements (Figure 2). Four values of in-plane diaphragm flexibility were studied. Each one is identified with the parameter $r_{ax} = 100E\, t/(E_0\, b)$, where E = modulus of elasticity, t = thickness, E_0 = reference modulus of elasticiy, and b = largest diaphragm side. Using $t = b/100$ for all cases, it results that $E = E_0\, r_{ax}$. Values of $E_0 = 200{,}000\ \text{kg/cm}^2$ and $r_{ax} = 1{\cdot}0$, $0{\cdot}1$, $0{\cdot}01$ and $0{\cdot}001$ were selected. For a system with $b = 10\ m$ for instance, $r_{ax} = 1{\cdot}0$ (considered here as the case of a rigid floor) could correspond to a 10 cm-thick reinforced concrete floor. By comparison, $r_{ax} = 0{\cdot}01$ could correspond to a 3/4″-thick wooden floor.

Each lateral-resisting element or frame is assumed to have a hysteretic behaviour defined by the Clough-Otani[11] model with $\alpha_c = 0{\cdot}4$. The yield force of each frame was determined using a typical static design process based on the application of a seismic force acting at a design eccentricity relative to the centre of rigidity. Design eccentricities e_d can be defined with the following formulas (see for instance Goel and Chopra[12]):

$$e_{d1} = \alpha e_s + \beta b = \alpha e_s + e_a \tag{1}$$

$$e_{d2} = \delta e_s - \beta b = \delta e_s - e_a \tag{2}$$

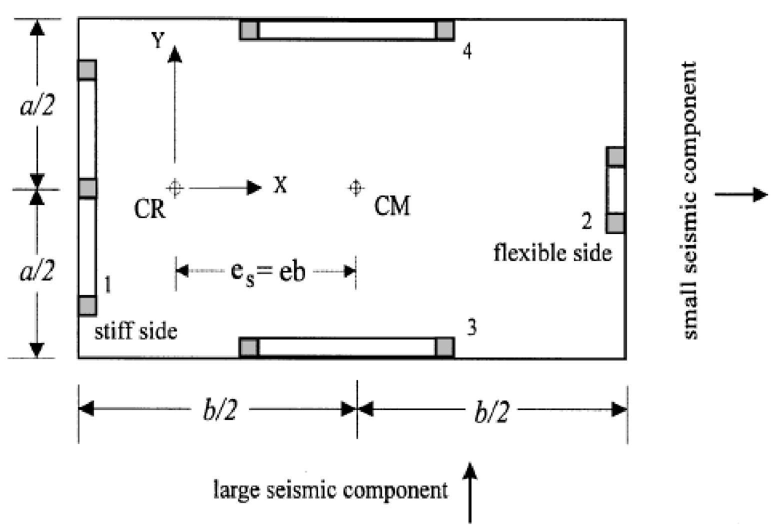

Figure 1. Structural model plan and directions of the seismic components

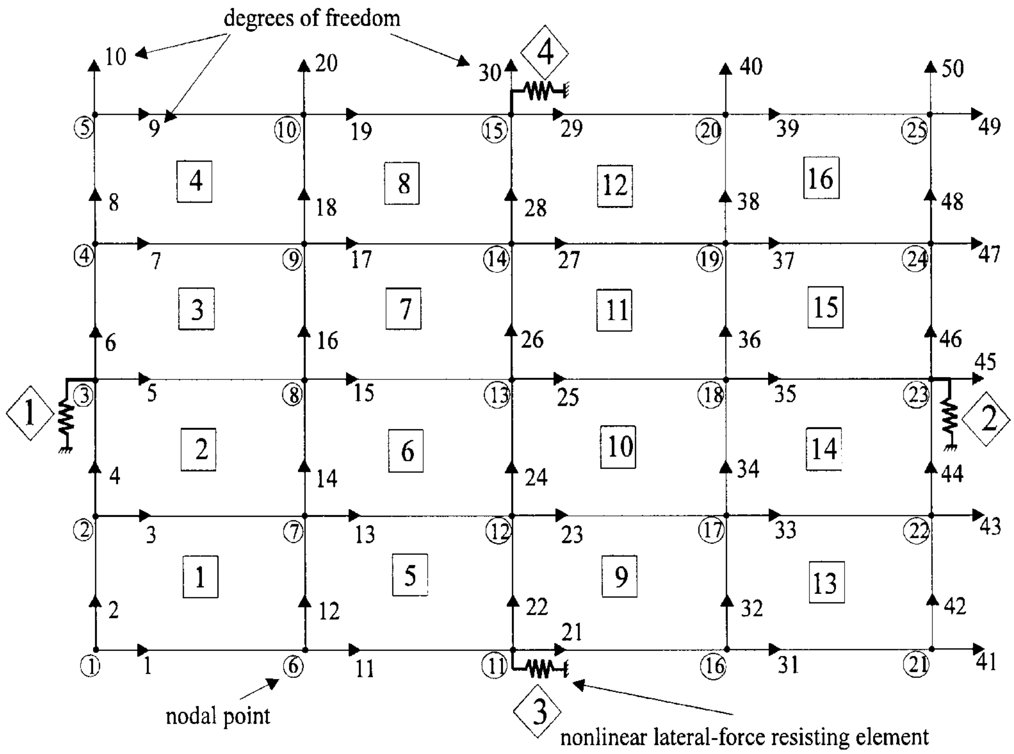

Figure 2. Diaphragm idealization using finite elements and non-linear frames (springs)

where e_a is the accidental eccentricity and α, β, and δ are torsion design factors. Here, $\alpha = 1{\cdot}5$, $\beta = 0{\cdot}0$, and $\delta = 0{\cdot}0$ were selected. The post-yield stiffness of the hysteretic model was assumed equal to 5 per cent of the initial stiffness for all frames.

To estimate the seismic design force, a spectral acceleration was obtained using a design spectrum like that presented by Newmark and Hall,[13] with a peak ground acceleration (PGA) of $1g$ for the Y direction. In order to be consistent with the seismic records used, a similar design spectrum was used for the X direction, but with a PGA of $0.64g$. This acceleration is the average of the PGAs for the scaled X-direction earthquake records. Both spectra are shown in Figure 3.

In order to simplify the analysis, the consistent mass matrix of the slab was diagonalized with the scheme recommended by Hinton *et al.*[14] A mass per unit volume of $1\ t - s^2/m^4$ was used for all cases. Rayleigh damping of 5 per cent was used for two selected frequencies (π and 60π). Results are reported for systems designed with three values of the seismic-force reduction factor ($R = 1$, 3, and 6), ten values of the initial lateral period ($T = 0{\cdot}2, 0{\cdot}4, \ldots, 2{\cdot}0$ s), and one eccentricity ($e = 0{\cdot}20$). Results obtained for $e = 0{\cdot}05$ are similar to those obtained for $e = 0{\cdot}20$ but are not shown here for brevity.

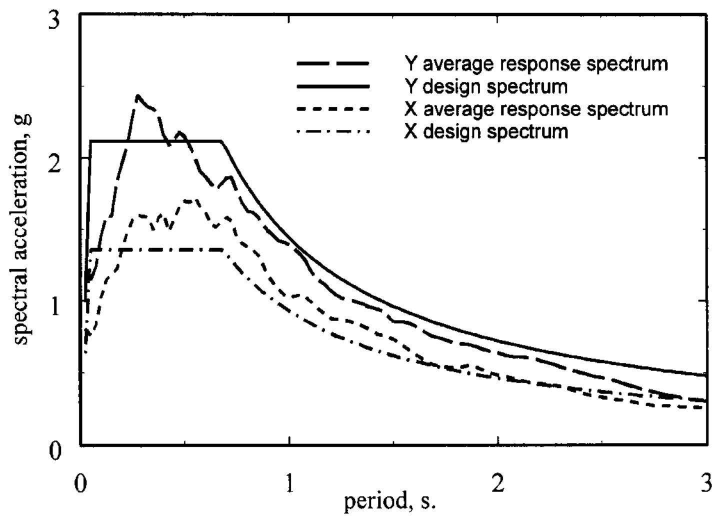

Figure 3. Average response spectra and design spectra

Table I. Earthquake ground motions

Earthquake	Station	Components	Duration used (s)	Peak recorded ground acceleration
Imperial Valley, May/18/1940	El Centro	S90W, S00E	20	0·21g, 0·35g
Kern County, July/21/1952	Santa Barbara	N42E, S48E	20	0·09g, 0·13g
México (Mich.), Sept/19/1985	La Union	EW, NS	60	0·15g, 0·17g
México (Mich.), Sept/19/1985	Papanoa	EW, NS	60	0·12g, 0·17g
San Salvador, Oct/10/1986	Nat. Inst. of Geo.	NS, EW	20	0·40g, 0·53g
San Salvador, Oct/10/1986	Geo. Res. Center	NS, EW	8	0·42g, 0·69g
Loma Prieta, Oct/17/1989	Corralitos	EW, NS	20	0·48g, 0·63g
Loma Prieta, Oct/17/1989	Presidio	NS, EW	20	0·10g, 0·20g
Northridge, Jan/17/1994	Sylmar/Hospital Park.	EW, NS	20	0·60g, 0·84g
Northridge, Jan/17/1994	S. Monica/City Hall G.	NS, EW	20	0·37g, 0·88g

SEISMIC RECORDS AND RESPONSE COMPUTATION

Each case was analysed for an ensemble of ten pairs of seismic records corresponding to firm soil (see Table I). For each ground motion, the component with largest PGA was scaled to $1g$ and was applied in the Y-direction (see Figure 1). The other component was applied in the perpendicular direction after being scaled with the same scale factor used for the Y-direction ground motion. Figure 3 shows the design spectra and the corresponding average response spectra for each direction. For all spectra, viscous damping equal to 5 per cent of the critical value was included.

The response computation was carried out using the constant-acceleration step-by-step method and the modified Newton–Raphson algorithm, both described in Chopra.[15] The response parameters used to evaluate the floor-flexibility effects are the averages of the peak displacement of the frame 1 (u_1), the peak displacement of the frame 2 (u_2), and the peak displacement of one of the X-direction frames (u_3 or u_4). In order to appreciate the response for several values of in-plane floor flexibility, these displacements are normalized with respect to the displacements computed with $r_{ax} = 1{\cdot}0$.

RESULTS

The averages of peak displacements for the *stiff-side* element (frame 1) of systems with $e = 0{\cdot}20$, normalized with respect to those computed with $r_{ax} = 1{\cdot}0$, are shown in Figure 4. As explained earlier, the most flexible diaphragm considered corresponds to $r_{ax} = 0{\cdot}001$, while the stiffest one is associated with the value $r_{ax} = 1{\cdot}0$. It is observed that increasing values of floor flexibility lead to decreasing *peak displacement averages* (PDAs) of frame 1 for almost all periods considered and for all values of R used. However, significant PDA increments (up to 50 per cent) can be observed for short-period systems (say $T \leqslant 0{\cdot}4$ s). Similar trends were observed for systems with $e = 0{\cdot}05$ (not shown here). It can be seen in Figure 4 that effects of in-plane floor flexibility appear to decrease for increasing values of both the seismic-force reduction factor R and the initial lateral period of vibration T.

Figures 5 and 6 show the averages of peak displacements (normalized with respect to those obtained with $r_{ax} = 1{\cdot}0$) for the *flexible-side* and the *X-direction* frames of systems with $e = 0{\cdot}20$. Results for $e = 0{\cdot}05$ (not shown here) are similar to those for $e = 0{\cdot}20$. In these figures, PDAs tend to reduce for increasing values of in-plane floor flexibility. Some significant PDA increments (up to 25%) are observed for systems with short initial periods. Again, it is clear that the effect of the in-plane floor flexibility is reduced with increasing values of R and T.

For the ensemble of earthquakes considered, the previous results indicate that frame displacements of systems with medium-to-large periods (say $T > 0{\cdot}4$ s) are reduced by the increase of in-plane floor flexibility. For systems with short periods (say $T \leqslant 0{\cdot}4$ s) increments (up to 50%) of frame displacements can be observed for the range of values considered. Because the design of all the systems considered was carried out with the assumption of a rigid in-plane floor, the yield displacement for each lateral resisting element is independent of the r_{ax} value. Thus, it is expected that the normalized displacement ductility demands will vary in the same way as the normalized displacements do.

In addition to the analysis of averages of frame peak displacements, taken across the set of ground motions, it is also interesting to observe the scatter of peak displacements. Figure 7, which is representative of the cases considered, shows the coefficients of variation of the normalized displacements of frame 1 for $R = 6{\cdot}0$. The scatter of these results indicate that the displacements due to individual earthquakes can significantly depart from mean values reported in Figures 4–6 for small floor flexibilities and periods.

For the cases considered, the effect of in-plane floor flexibility decreases for increasing values of the seismic-force reduction factor R and the initial lateral period of vibration T. This behaviour, which is consistent with the observation of Saffarini and Qudaimat[7] (noted in the introduction) is reasonable because increments of either R or T cause a reduction on the effective lateral system

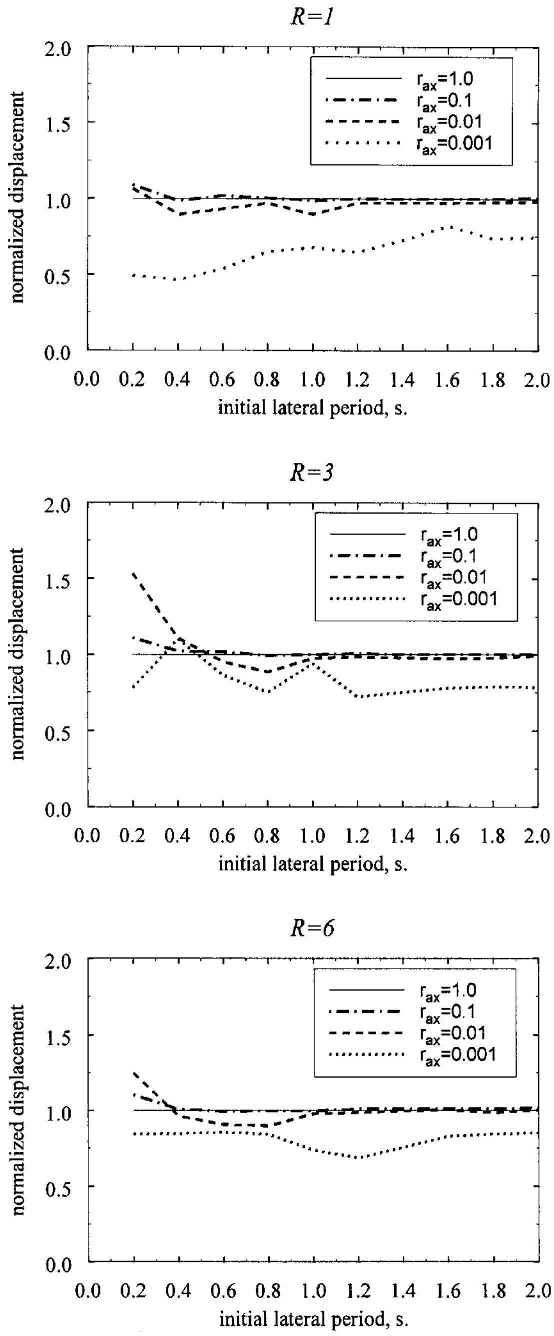

Figure 4. Normalized displacements for the *stiff*-side element (frame 1) for $e = 0{\cdot}20$

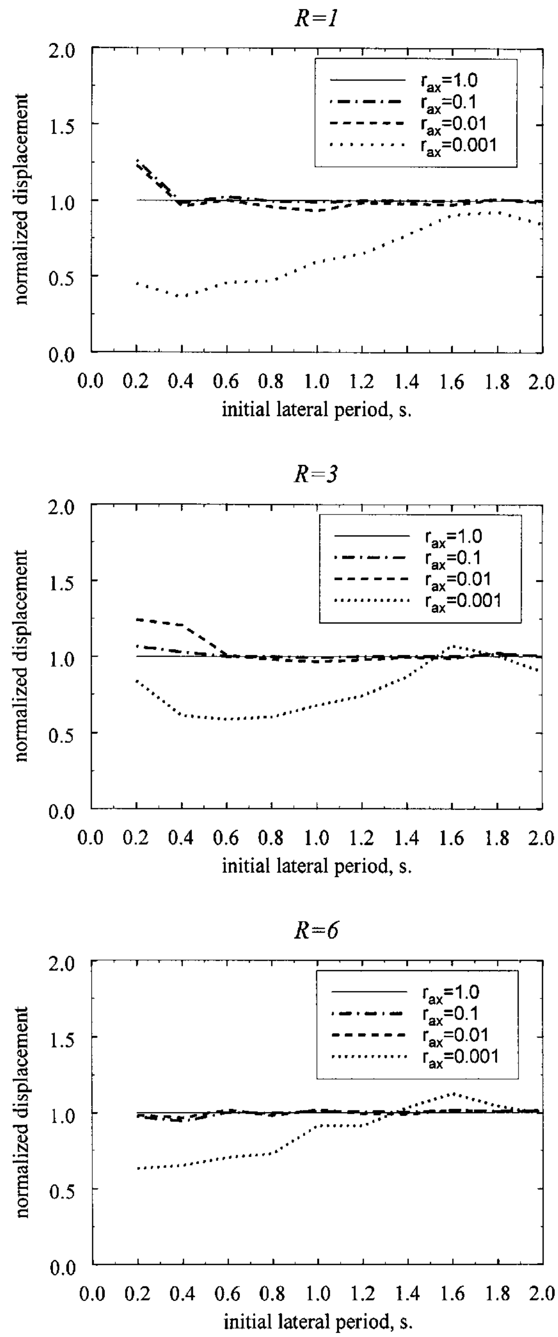

Figure 5. Normalized displacements for the *flexible*-side (frame 2) element for $e = 0{\cdot}20$

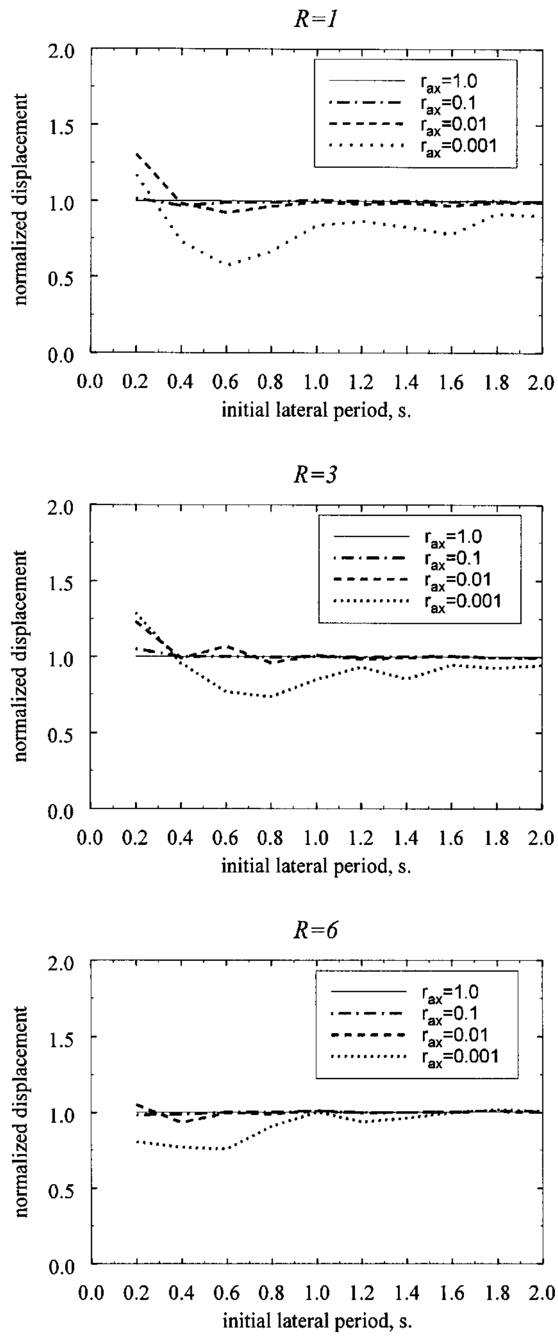

Figure 6. Normalized displacements for *X-direction* elements (frames 3 or 4) for $e = 0{\cdot}20$

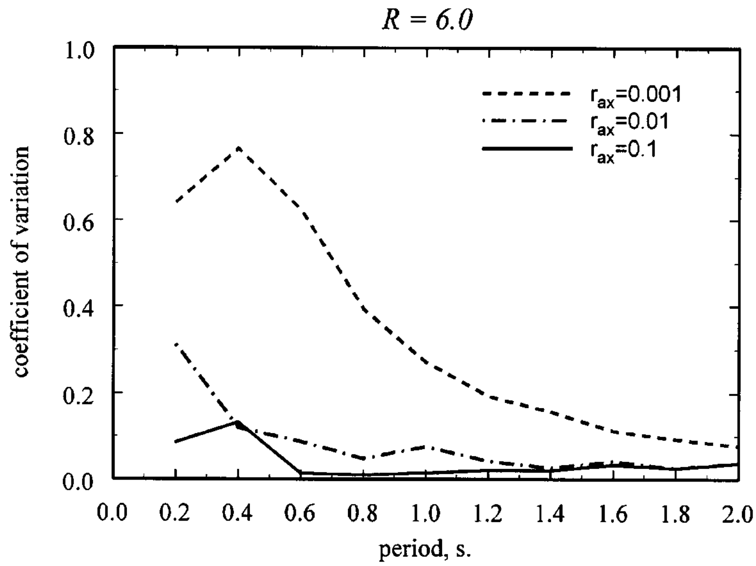

Figure 7. Coefficients of variation for peak displacements of frame 1

stiffness which, in turn, increases the ratio of the in-plane floor stiffness to the lateral-load resisting system stiffness.

A simple explanation of both the increment of frame displacements for small periods and the decrement for medium-to-large periods can be offered by considering: (1) the shape of the acceleration response spectra and (2) the period elongation caused by the in-plane floor flexibility. For short periods, the in-plane floor flexibility increases the effective lateral period, causing larger spectral accelerations and displacements. For medium-to-large initial periods, the period elongation leads to smaller spectral accelerations than those computed on the basis of a rigid-floor system.

CONCLUSIONS

The analytical study reported here on simple torsionally unbalanced systems subjected to bidirectional firm-soil earthquake records leads to the following conclusions. The peak displacement averages (PDAs) of lateral-resisting elements decrease for increasing in-plane floor flexibilities of systems with medium-to-large initial lateral periods ($T > 0.4$ s). The PDA of these elements increases (up to 50 per cent higher) for systems with short initial periods ($T \leqslant 0{\cdot}4$ s). Results indicate that the scatter of peak displacements decreases with increasing values of both the initial lateral period and the in-plane floor stiffness. In all cases, the in-plane floor flexibility effect decreases for increasing values of the seismic-force reduction factor R and the initial lateral period of vibration T.

REFERENCES

1. W. K. Tso and T. J. Zhu, 'Design of torsionally unbalanced structural systems based on code provisions I: ductility demand', *Earthquake Engng. Struct. Dyn.* **21**, 609–627 (1992).

2. J. E. Goldberg and E. D. Herness, 'Vibration of multistory buildings considering floor and wall deformations', *Bull. Seismol. Soc. Am.* **55**, 181–200 (1965).
3. R. Shepherd and R. Donald, 'The influence of in-plane floor flexibility on the normal mode properties of buildings', *J. Sound Vib*. **5**, 29–36 (1967).
4. S. K. Jain, 'Seismic response of buildings with flexible floors', *J. Engng Mech. ASCE* **110**, 125–129 (1984).
5. S. K. Jain and P. C. Jennings, 'Analytical models for low-rise buildings with flexible floor diaphragms', *Earthquake Engng. Struct. Dyn.* **13**, 225–241 (1985).
6. S. Kunnath, N. Panahshahi and A. Reinhorn, 'Seismic response of RC buildings with inelastic floor diaphragms', *J. Struct. Engng. ASCE* **117**, 1218–1237 (1991).
7. H. Saffarini and M. Qudaimat, 'In-plane floor deformations in RC structures', *J. Struct. Engng. ASCE* **118**, 3089–3102 (1992).
8. S. H. Ju and M. C. Lin, 'Comparison of building analyses assuming rigid or flexible floors', *J. Struct. Engng. ASCE* **125**, 25–39 (1999).
9. A. Tena-Colunga and D. P. Abrams, 'Seismic behavior of structures with flexible diaphragms', *J. Struct. Engng. ASCE* **122**, 439–445 (1996).
10. F. S. Yeomans and F. Dávila, 'Investigación sobre diafragmas en el ITESM - frente I: diseño de mesa de carga y prueba de diafragma testigo', *Rep. Final Dep. Ing. Civil.* ITESM (1997).
11. S. Otani, 'Hysteresis models of reinforced concrete for earthquake response analysis', *J. Faculty Engng.* (*B*), University of Tokyo **36**, 125–159 (1981).
12. R. Goel and A. K. Chopra, 'Inelastic seismic response of one-story, asymmetric-plan systems', *EERC Report No. 90/14*, Earthquake Eng. Res. Center, University of California at Berkeley, CA, 1990.
13. N. M. Newmark and W. J. Hall, *Earthquake Spectra and Design*, Earthquake Eng. Res. Inst., Berkeley, CA, 1982.
14. E. Hinton, A. Rock and O. C. Zienkiewicz, 'A note on mass lumping and related processes in the finite element method', *Earthquake Eng. Struct. Dyn.* **4**, 245–249 (1976).
15. A. K. Chopra, *Dynamics of Structures — Theory and Applications to Earthquake Engineering*, Prentice-Hall, Englewood Cliffs, NJ, 1995.

